Chapter 11 Practice Exercises (*Optional)

11-01 3-D Coordinate System

Graph the following points.

1. $A(3, 1, -2)$ and $B(-3, 2, 2)$
2. $C(0, -1, 0)$ and $D(2, -2, 4)$
3. $E(-3, 0, 1)$ and $F(4, 1, 0)$
4. Graph the line passing through $(-4, 2, 3)$ and $(2, -4, -3)$.

Find the distance between the two points.

5. $A(3, 1, -2)$ and $B(-3, 2, 2)$
6. $C(0, -1, 0)$ and $D(2, -2, 4)$
7. $E(-3, 0, 1)$ and $F(4, 1, 0)$

Find the midpoint between the two points.

8. $A(3, 1, -2)$ and $B(-3, 2, 2)$
9. $C(0, -1, 0)$ and $D(2, -2, 4)$
10. $E(-3, 0, 1)$ and $F(4, 1, 0)$

Write the equation of the sphere with the given properties.

11. Center at $(2, -3, 1)$ and radius 5
12. Center at $(0, 2, -1)$ and a point on the sphere is $(2, 3, 4)$
13. The ends of the diameter are at $(-1, 4, 0)$ and $(3, 8, -6)$.

Graph the sphere.

14. $(x + 1)^2 + (y - 2)^2 + z^2 = 9$
15. $(x - 3)^2 + y^2 + (z + 1)^2 = 16$

Mixed Review

16. (10-08) What is the probability of randomly guessing the correct answers on a 5 question true/false quiz?
17. (10-07) You have 16 model ships to display on a shelf. How many different orders can you display 10 of them because that is all the space you have on the shelf?
18. (10-02) Evaluate $\sum_{i=1}^{20}(i^3 + i)$.
19. (6-05) Evaluate $\langle 3, 4\rangle \cdot \langle -2, 2\rangle$.
20. (6-03) Given $\vec{m} = \langle 2, -3\rangle$ and $\vec{n} = \langle -1, 6\rangle$, find $2\vec{m} + \vec{n}$.

11-02 Vectors In Space

Let $\vec{r} = \langle 2, 4, -1\rangle$, $\vec{s} = \langle -3, 0, -2\rangle$, and $\vec{t} = \langle 1, -5, 4\rangle$. Evaluate the following.

1. $\vec{r} + \vec{s}$
2. $\vec{t} - \vec{r}$
3. $3\vec{r}$
4. $2\vec{s} + 3\vec{t} - \vec{r}$
5. $\|\vec{r}\|$
6. $\|\vec{t}\|$
7. Unit vector in direction of $\vec{r}$
8. Unit vector in direction of $\vec{t}$
9. $\vec{r} \cdot \vec{s}$
10. $\vec{t} \cdot \vec{r}$
11. Find the angle between $\vec{r}$ and $\vec{s}$
12. Find the angle between $\vec{t}$ and $\vec{s}$

Determine if the vectors are parallel, perpendicular, or neither.

13. $\langle 3, -1, \frac{1}{2}\rangle$ and $\langle -2, -3, 6\rangle$
14. $\langle 36, -1, \frac{1}{2}\rangle$ and $\langle 12, -\frac{1}{3}, \frac{1}{6}\rangle$
15. Determine whether the points $G(2, -1, 3)$, $H(4, 1, 5)$, and $J(8, 5, 9)$ are collinear.

Mixed Review

16. (11-01) Find the distance between $(3, 1, -2)$ and $(-1, 0, 0)$.
17. (11-01) Find the equation of the sphere with center $(1, 3, -1)$ and radius of 5.
18. (10-06) Use the binomial theorem to expand $(x - 2y)^4$.
19. (9-05) Find the determinant of $\begin{bmatrix} 2 & 1 & 0 \\ -2 & 3 & -1 \\ 0 & 4 & -3 \end{bmatrix}$.
20. (7-09) Find the polar equation of a parabola with focus at the origin and directrix $y = -4$.

11-03 Cross Products

Given that $\vec{m} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{n} = \hat{i} - 3\hat{k}$, evaluate the following.

1. $\vec{m} \times \vec{n}$
2. $2(\vec{n}) \times \vec{m}$
3. $\vec{n} \times \vec{n}$
4. $(\vec{n} \times \vec{m}) \times \vec{m}$
5. $\vec{m} \cdot (\vec{m} \times \vec{n})$

Find the cross product of the vectors.

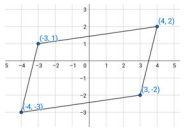
6. $\langle 3, 2, -4\rangle$ and $\langle 0, -1, 2\rangle$
7. $\langle -1, 4, 10\rangle$ and $\langle 2, 1, 3\rangle$

Find a unit vector that orthogonal to $\vec{u}$ and $\vec{v}$.

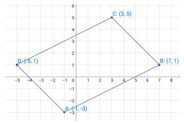
8. $\vec{u} = \langle 3, -1, 1\rangle$ and $\vec{v} = \langle -2, 0, 1\rangle$
9. $\vec{u} = \langle 0, 2, 0\rangle$ and $\vec{v} = \langle -1, 1, 1\rangle$
10. $\vec{u} = \langle 10, 15, -5\rangle$ and $\vec{v} = \langle -1, 3, -2\rangle$

Use cross products to find the area of the parallelogram.

11.



12.



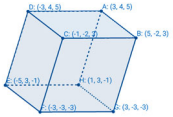
Evaluate the triple scalar product of the three vectors.

13. $\vec{u} = 2\hat{i} + 5\hat{j} - \hat{k}$
$\vec{v} = \hat{i} + \hat{k}$
$\vec{w} = -3\hat{i} - 2\hat{j} + 2\hat{k}$

14. $\vec{a} = \langle 2, 5, 1\rangle$
$\vec{b} = \langle 0, -2, 4\rangle$
$\vec{c} = \langle 0, 2, 0\rangle$

Find the volume of the parallelepiped.

15.



Problem Solving

16. The Docklands building in Hamburg, Germany is shaped like a parallelogram. If a grid were covered the side of the building, the vertices are (0, 0), (86, 0), (49, 55), and (135, 55). Use a cross product to find the area of the side of the building.

Mixed Review

17. (11-02) Find the angle between $\langle 3, 4, 0\rangle$ and $\langle -2, 0, 1\rangle$.
18. (11-02) Evaluate $\langle 3, 4, 0\rangle - \langle -2, 0, 1\rangle$.
19. (11-01) Find the distance between $(3, 4, 0)$ and $(-2, 0, 1)$.
20. (10-08) What is the probability of rolling two dice and having both results be 4 or less?

11-04 Lines and Planes in Space

Find the specified equations of a line through the given points. (Use the first point for (x_1, y_1, z_1).)

1. Parametric form through $(2, 3, -2)$ and $(4, 0, -1)$
2. Parametric form through $(-3, 0, 2)$ and $(-2, 5, 4)$
3. Symmetric form through $(-4, 5, -2)$ and $(0, -1, -5)$

Find the general form of the equation of the plane containing the given points.

4. $(1, 0, 4)$, $(3, -2, 0)$, and $(0, -2, 1)$
5. $(3, 2, 1)$, $(4, -2, -1)$, and $(0, 0, 3)$
6. $(4, 2, -4)$, $(5, -2, 0)$, and $(9, -2, 1)$

Find the angle between the two planes.

7. $2x - 3y + z - 4 = 0$ and $x + y + z + 1 = 0$
8. $-x + 3y = 6$ and $2x - y + 3z = -5$
9. $3x + 4z + 3 = 0$ and $4y - 2z - 7 = 0$

Find the distance from the point to the plane.

10. $2x - 3y + z - 4 = 0$; $(3, 2, -1)$
11. $-x + 3y = 6$; $(-2, 5, 0)$
12. $3x + 4z + 3 = 0$; $(1, 1, 1)$

Sketch a graph of the plane.

13. $3x + 2y + 6z - 18 = 0$
14. $5x + 4y - 2z = 20$
15. $x - y + z = 6$

Mixed Review

16. (11-03) Find a vector orthogonal to $\langle 2, -1, 1\rangle$ and $\langle 1, 3, 3\rangle$.
17. (11-03) Find $\langle 3, 0, -3\rangle \times \langle 0, -4, 2\rangle$.
18. (11-02) Find a unit vector in the direction of $\langle 3, 2, 2\sqrt{3}\rangle$.
19. (11-01) Find the midpoint between $(2, 0, -4)$ and $(-8, 10, 36)$.
20. (8-06) Find the maximum value of $z = 2x - y$ given the constraints $\begin{cases} x + y \le 10 \\ x \ge 1 \\ y \ge 2 \end{cases}$

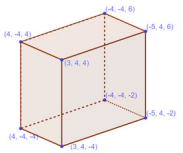
11-Review

Take this test as you would take a test in class. When you are finished, check your work against the answers. On this assignment round your answers to three decimal places unless otherwise directed.

1. Plot the points $A(1, -2, 3)$, $B(0, 3, -5)$, and $C(-2, 0, 2)$

Use the points $A(1, -2, 3)$, $B(0, 3, -5)$, and $C(-2, 0, 2)$. Let $\vec{u}$ be the vector from A to B and $\vec{v}$ be the vector from B to C.

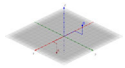
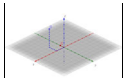
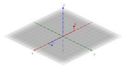
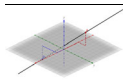
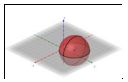
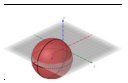
2. Is the triangle formed isosceles?
3. Find the midpoint of segment AC.
4. Find the standard form of the equation of a sphere with B as the center and C on the surface of the sphere.
5. Find the parametric equations for the line passing through A and B.
6. Find the general form of the equation of the plane passing through the points A, B, and C.
7. Find the distance between point A and the plane $x + 2y - z - 4 = 0$.
8. Write $\vec{u}$ and $\vec{v}$ in component form.
9. Find $\|\vec{u}\|$.
10. Find a unit vector in the direction of $\vec{u}$.
11. Find $\vec{u} \cdot \vec{v}$.
12. Find $\vec{u} \times \vec{v}$.
13. Find $\vec{u} \times \vec{u}$.
14. Find the angle between $\vec{u}$ and $\vec{v}$.
15. Use vectors to find the area of the parallelogram with vertices $(-2, -2)$, $(4, -1)$, $(0, 5)$, and $(6, 6)$.
16. Determine whether $\vec{n} = 2\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{m} = \hat{i} + 2\hat{j} + \hat{k}$ are parallel, orthogonal, or neither.
17. Determine whether $\vec{n} = 5\hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{m} = -3\hat{i} + \hat{j} + 2\hat{k}$ are parallel, orthogonal, or neither.
18. Find the volume of the parallelepiped.



19. Plot the intercepts and graph the plane $4x + 4y + 3z - 12 = 0$
20. Plot the intercepts and graph the plane $6x + 3y - 4z + 12 = 0$

Answers

11-01

1. 
2. 
3. 
4. 
5. $\sqrt{53}$
6. $\sqrt{21}$
7. $\sqrt{51}$
8. $\left(0, \frac{3}{2}, 0\right)$
9. $\left(1, -\frac{3}{2}, 2\right)$
10. $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$
11. $(x-2)^2 + (y+3)^2 + (z-1)^2 = 25$
12. $x^2 + (y-2)^2 + (z+1)^2 = 30$
13. $(x-1)^2 + (y-6)^2 + (z+3)^2 = 17$
14. 
15. 
16. 0.03125
17. 2.91×10^{10}
18. 44310
19. 2
20. $\langle 3, 0\rangle$

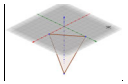
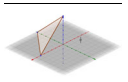
11-02

1. $\langle -1, 4, -3\rangle$
2. $\langle -1, -9, 5\rangle$
3. $\langle 6, 12, -3\rangle$
4. $\langle -5, -19, 9\rangle$
5. $\sqrt{21}$
6. $\sqrt{42}$
7. $\left\langle \frac{2\sqrt{21}}{21}, \frac{4\sqrt{21}}{21}, -\frac{\sqrt{21}}{21} \right\rangle$
8. $\left\langle \frac{\sqrt{42}}{42}, -\frac{5\sqrt{42}}{42}, \frac{2\sqrt{42}}{21} \right\rangle$
9. -4
10. -22
11. $104°$
12. $118°$
13. Perpendicular
14. Parallel
15. Collinear
16. $\sqrt{21}$
17. $(x-1)^2 + (y-3)^2 + (z+1)^2 = 25$
18. $x^4 - 8x^3y + 24x^2y^2 - 32xy^3 + 16y^4$
19. -16
20. $r = \frac{4}{1 - \sin\theta}$

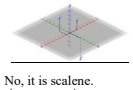
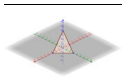
11-03

1. $\langle -3, 4, -1\rangle$
2. $\langle 6, -8, 2\rangle$
3. $\langle 0, 0, 0\rangle$
4. $\langle 7, 8, 11\rangle$
5. 0
6. $\langle 0, -6, -3\rangle$
7. $\langle 2, 23, -9\rangle$
8. $\left\langle -\frac{\sqrt{30}}{30}, -\frac{\sqrt{30}}{6}, -\frac{\sqrt{30}}{15} \right\rangle$
9. $\left\langle \frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2} \right\rangle$
10. $\left\langle -\frac{8\sqrt{115}}{115}, \frac{\sqrt{115}}{23}, \frac{9\sqrt{115}}{115} \right\rangle$
11. 27
12. 48
13. -19
14. -16
15. 204
16. 4730 m^2
17. $122.46°$
18. $\langle 5, 4, -1\rangle$
19. $\sqrt{42}$
20. $\frac{4}{9}$

11-04

1. $\begin{cases} x = 2 + 2t \\ y = 3 - 3t \\ z = -2 + t \end{cases}$
2. $\begin{cases} x = -3 + t \\ y = 5t \\ z = 2 + 2t \end{cases}$
3. $\frac{x+4}{4} = \frac{y-5}{-6} = \frac{z+2}{-3}$
4. $2x - 10y + 6z - 26 = 0$
5. $12x - 4y + 14z - 42 = 0$
6. $4x - 15y - 16z - 50 = 0$
7. $90°$
8. $\approx 65.0°$
9. $\approx 69.0°$
10. $\frac{5\sqrt{14}}{14}$
11. $\frac{11\sqrt{10}}{10}$
12. 2
13. 
14. 
15. 
16. $\langle -6, -5, 7\rangle$
17. $\langle -12, -6, -12\rangle$
18. $\left\langle \frac{3}{5}, \frac{2}{5}, \frac{2\sqrt{3}}{5} \right\rangle$
19. $\langle -3, 5, 16\rangle$
20. 14 at (8, 2)

11-Review

1. 
2. No, it is scalene.
3. $\left(-\frac{1}{2}, -1, \frac{5}{2}\right)$
4. $x^2 + (y-3)^2 + (z+5)^2 = 62$
5. $\begin{cases} x = 1 - t \\ y = -2 + 5t \\ z = 3 - 8t \end{cases}$
6. $11x + 23y + 13z - 4 = 0$
7. $\frac{5\sqrt{6}}{3}$
8. $\vec{u} = \langle -1, 5, -8\rangle$, $\vec{v} = \langle -2, -3, 7\rangle$
9. $3\sqrt{10}$
10. $\left\langle -\frac{\sqrt{10}}{30}, \frac{\sqrt{10}}{6}, -\frac{4\sqrt{10}}{15} \right\rangle$
11. -69
12. $\langle 11, 23, 13\rangle$
13. $\langle 0, 0, 0\rangle$
14. 157.47°
15. 40
16. orthogonal
17. neither
18. 512
19. 
20.